

Higher dimensional operator corrections to the goldstino Goldberger–Treiman vertices

Taekoon Lee^a

Center for Theoretical Physics, Seoul National University, Seoul 151-742, Korea

Received: 22 December 1998 / Revised version: 1 July 1999 /
Published online: 17 February 2000 – © Springer-Verlag 2000

Abstract. The goldstino–matter interactions given by the Goldberger–Treiman relations can receive higher dimensional operator corrections of $O(q^2/M^2)$, where M denotes the mass of the mediators through which SUSY breaking is transmitted. These corrections in the gauge mediated SUSY breaking models arise from loop diagrams, and an explicit calculation of such corrections is presented. It is emphasized that the Goldberger–Treiman vertices are valid only below the mediator scale, and at higher energies goldstinos decouple from the MSSM fields. The implication of this fact for gravitino cosmology in GMSB models is mentioned.

The interactions of the light gravitino with matter are dominated by the spin 1/2 longitudinal component of the gravitino, which is essentially the goldstino eaten by the gravitino via the supersymmetric Higgs mechanism. At energies much higher than the gravitino mass, the supersymmetric version of the equivalence theorem [1, 2] allows one to replace the gravitino with the goldstino. The low energy interactions of a goldstino with matter fields, which in this letter are assumed to be the fields in the minimal supersymmetric standard model (MSSM), are completely fixed model-independently by the so called goldstino Goldberger–Treiman vertices [1, 3] which depend on the mass splittings of the superpartners. This is similar to the Goldberger–Treiman relations in pion–nucleon interactions which also depend on the chiral symmetry breaking parameters, namely, the nucleon masses.

At high energies the goldstino Goldberger–Treiman interactions are expected to get corrections of $O(q^2)$, where q^2 denotes generic Lorentz invariants of the external momenta. At first glance, one might think that this correction is suppressed by $1/F$, where F is the goldstino decay constant, in analogy to the correction in the pion–nucleon interaction which is of $O(q^2/f_\pi^2)$. However, unlike the pion–nucleon case in which there is only one fundamental scale, namely, f_π , there can be multiple scales in realistic SUSY models, so it is possible that the correction is suppressed by $1/M_X^2$, where M_X is an intermediate scale between the MSSM scale and $F^{1/2}$. If this is indeed the case, the corrections can be much larger than a naive expectation based on the analogy to the Goldberger–Treiman relations in hadron physics. In this letter, we show that the corrections to the goldstino Goldberger–Treiman couplings can be suppressed by an intermediate scale by an explicit

calculation of such corrections in gauge mediated SUSY breaking (GMSB) models [4].

In GMSB models, SUSY breaking occurs in a hidden sector and is transmitted to the MSSM sector through gauge interactions between mediators and the MSSM fields. The mediator scale in these models can be much lower than the SUSY breaking scale. Because the SUSY breaking occurs in the hidden sector, there is no tree level coupling between the goldstino and the MSSM fields, and the Goldberger–Treiman vertices are induced through loop diagrams. Since the goldstino–matter interaction arises from loops, it becomes clear that the correction to the Goldberger–Treiman vertices can be $O(q^2/M^2)$, where M denotes the mass of the mediators that go through the loop diagrams, unless there is an exact cancellation among the diagrams. We shall see that such a cancellation does not occur in GMSB models.

We consider a GMSB model in which there is a gauge singlet superfield S through which the hidden sector and the visible sector are connected. S communicates to the MSSM fields through interactions with a set of mediators $\{q_{1i}, q_{2i}\}$ via the superpotential

$$\mathcal{L}_w = h \sum_i^{N_q} S q_{1i} q_{2i}, \quad (1)$$

where N_q denotes the number of mediators, and h is a coupling constant. Note that q_{1i}, q_{2i} carry opposite standard model gauge quantum numbers, respectively. For our purpose, the details of the interaction between S and the hidden sector fields are not needed; The only requirement is that the VEVs $\langle F_s \rangle$ and $\langle S \rangle$ are nonvanishing.

For simplicity we first consider SUSY QED for the MSSM sector, since the corrections to the Goldberger–Treiman vertices in non-abelian gauge theories turn out

^a e-mail: tlee@ctp.snu.ac.kr

to be identical to those of the abelian case. The goldstino Goldberger–Treiman vertices between a massless Weyl fermion (ψ), sfermion (ϕ) and gauge boson (A_μ), gaugino (λ) are given by [1,5]

$$\begin{aligned} \mathcal{L}_{\text{GT}} = & \frac{m_\phi^2}{F} \chi \psi \phi^* + \frac{im_\lambda}{\sqrt{2}F} \chi \sigma^{\mu\nu} \lambda F_{\mu\nu} \\ & - \frac{em_\lambda}{\sqrt{2}F} \phi^* \phi \chi \lambda + \text{h.c.}, \end{aligned} \quad (2)$$

where χ denotes the goldstino, m_ϕ , m_λ are the sfermion and gaugino masses, respectively, and e is the gauge coupling. Throughout this paper we follow the convention for spinors and metric given in [6], except that our gaugino λ is related to the gaugino in [6] by $\lambda = -i\lambda_{WB}$. Note that the metric in this convention is $\text{Diag}(-1, 1, 1, 1)$.

The interaction lagrangian in SUSY QED is given by

$$\begin{aligned} \mathcal{L}_{\text{QED}} = & -eA_\mu \psi \sigma^\mu \bar{\psi} + ieA_\mu (\phi^* \partial^\mu \phi - \partial^\mu \phi^* \phi) \\ & - \sqrt{2}e(\phi^* \psi \lambda + \phi \bar{\psi} \bar{\lambda}) - \frac{e^2}{2} (\phi^* \phi)^2 \\ & - e^2 A_\mu A^\mu \phi^* \phi, \end{aligned} \quad (3)$$

and the couplings between the mediators and the SUSY QED fields are given by

$$\begin{aligned} \mathcal{L}_1 = & -eA_\mu [\Psi_{1i} \sigma^\mu \bar{\Psi}_{1i} - \Psi_{2i} \sigma^\mu \bar{\Psi}_{2i} \\ & + i(\Phi_{1i}^* \partial^\mu \Phi_{1i} - \partial^\mu \Phi_{1i}^* \Phi_{1i} - \Phi_{2i}^* \partial^\mu \Phi_{2i} + \partial^\mu \Phi_{2i}^* \Phi_{2i})] \\ & - \sqrt{2}e(\Phi_{1i}^* \Psi_{1i} \lambda - \Phi_{2i}^* \Psi_{2i} \lambda + \text{h.c.}) \\ & - e^2 \phi^* \phi (\Phi_{1i}^* \Phi_{1i} - \Phi_{2i}^* \Phi_{2i}), \end{aligned} \quad (4)$$

where the last term arises from the D term.

When SUSY is broken in the hidden sector and F_S develops a nonzero VEV, a mixing occurs between the spin half component ψ_S of the chiral field S and the goldstino from the hidden sector. Due to the mixing, the true goldstino has a ψ_S component given by

$$\chi = -\frac{F_S}{F} \psi_S + \dots, \quad (5)$$

where the ignored terms involve only hidden sector fermions. For small F_S/F , which is assumed in this letter, the above relation can be inverted, giving

$$\psi_S = \frac{F_S}{F} \chi + \dots \quad (6)$$

Now using the superpotential (1) we obtain the interaction between the goldstino and the mediators as

$$\mathcal{L}_2 = h \frac{F_S}{F} (\Phi_{1i} \Psi_{2i} \chi + \Phi_{2i} \Psi_{1i} \chi + \text{h.c.}). \quad (7)$$

Note that this interaction is nothing but the Goldberger–Treiman vertex in the mediator sector since hF_S is the mass squared splitting of the mediators.

From the above interactions, it is easy to see that the Goldberger–Treiman vertices (2) arise from loop diagrams. The $\phi^* \psi \chi$ vertex in (2) comes from the two-loop

diagrams (Fig. 1) and the $A_\mu \lambda \chi$, $\phi^* \phi \lambda \chi$ vertices arise from the one-loop diagrams in Fig. 2 and Fig. 3, respectively. Phenomenologically, at high energies the dim-5 operators in the Goldberger–Treiman vertices are more interesting since the cross sections due to the dim-4 operator are always suppressed by $O(m^2/s)$, where m denotes the soft masses in MSSM and s is the c.m. energy squared, compared to those from the dim-5 operators. We therefore consider the higher dimensional operator corrections only for the dim-5 operators.

Let us first consider the $A_\mu \lambda \chi$ vertex in (2). We first assume that the mass splitting between the superpartners in the mediator sector is much smaller than the mediator mass. This requires

$$hS^2 \gg F_S. \quad (8)$$

Then, as mentioned, this vertex arises from the diagrams in Fig. 2. There are other one-loop diagrams; however, they are suppressed by $O(F_S/hS^2)$ compared to those in Fig. 2 and so can be ignored. It is straightforward to calculate the diagrams. First the diagram (1) gives

$$\begin{aligned} \mathcal{A}_1 = & i \frac{\sqrt{2} h e^2 N_q M F_S}{F} \\ & \times \int \prod_i^3 d^4 p_i \tilde{\lambda}(p_1) \tilde{\chi}(p_2) \tilde{A}_\mu(p_3) (2\pi)^4 \\ & \times \delta^4 \left(\sum_i^3 p_i \right) I_\mu^{(1)}(p_1, p_2, M), \end{aligned} \quad (9)$$

where

$$\begin{aligned} I_\mu^{(1)}(p_1, p_2, M) = & \frac{i}{16\pi^2} \int \prod_i^3 dx_i \delta \left(\sum_1^3 x_i - 1 \right) \\ & \times \left[\frac{(1-2x_1)p_{1\mu} - (1-2x_2)p_{2\mu}}{M^2 - t^2 + x_1 p_1^2 + x_2 p_2^2} \right], \end{aligned} \quad (10)$$

with

$$t_\mu = (x_1 p_1 - x_2 p_2)_\mu. \quad (11)$$

Here x_i are the Feynman parameters, M is the mediator mass

$$M = h\langle S \rangle, \quad (12)$$

and the Fourier transform is defined as

$$\tilde{f}(p) = \int \frac{d^4 p}{(2\pi)^4} f(x) e^{ip \cdot x}. \quad (13)$$

From the diagrams (2) and (3) we get

$$\begin{aligned} \mathcal{A}_2 = & -i \frac{\sqrt{2} h e^2 N_q M F_S}{F} \int \prod_i^3 d^4 p_i \\ & \times \tilde{\lambda}(p_1) \sigma^\mu \bar{\sigma}^\nu \tilde{\chi}(p_2) \tilde{A}_\mu(p_3) (2\pi)^4 \\ & \times \delta^4 \left(\sum_i^3 p_i \right) I_\nu^{(2)}(p_1, p_2, M), \end{aligned} \quad (14)$$

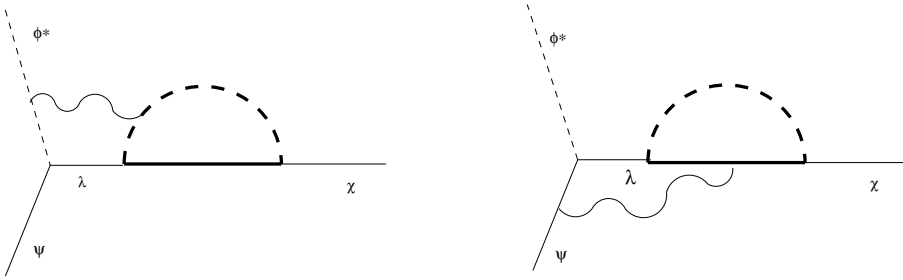


Fig. 1. Examples of two-loop diagrams that give rise to the $\phi^*\psi\chi$ Goldberger–Treiman vertex. Wavy lines denote gauge bosons and the thick solid and dashed lines denote fermionic and bosonic mediators, respectively

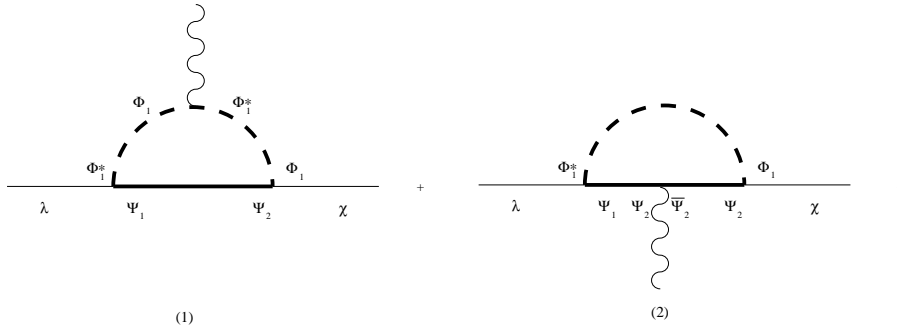


Fig. 2. Diagrams that give rise to the $A_\mu\lambda\chi$ Goldberger–Treiman vertex. Wavy lines denote gauge bosons and the thick solid and dashed lines denote fermionic and bosonic mediators, respectively. q_1 and q_2 denote mediators

where

$$I_\nu^{(2)}(p_1, p_2, M) = -\frac{i}{16\pi^2} \int \prod_i^3 dx_i \delta\left(\sum_1^3 x_i - 1\right) \times \left[\frac{(t + p_2)_\nu}{M^2 - t^2 + x_1 p_1^2 + x_2 p_2^2} \right], \quad (15)$$

and

$$\mathcal{A}_3 = -i \frac{\sqrt{2} h e^2 N_q M F_S}{F} \int \prod_i^3 d^4 p_i \times \tilde{\lambda}(p_1) \sigma^\nu \bar{\sigma}^\mu \tilde{\chi}(p_2) \tilde{A}_\mu(p_3) (2\pi)^4 \times \delta^4\left(\sum_i^3 p_i\right) I_\nu^{(3)}(p_1, p_2, M), \quad (16)$$

with

$$I_\nu^{(3)}(p_1, p_2, M) = -I_\nu^{(2)}(p_2, p_1, M). \quad (17)$$

The three other diagrams obtained from diagrams (1), (2) and (3) by exchanging the mediators $q_{1i} \leftrightarrow q_{2i}$ give identical amplitudes to their corresponding diagrams. Now for small external momenta compared to the mediator mass,

we can expand in $1/M^2$ the denominators in $I^{(i)}$ and integrate over x_i explicitly. Adding the six diagrams, this gives to $O(p_i \cdot p_j/M^2)$

$$\mathcal{A} = \sum_1^6 \mathcal{A}_i = -\frac{\sqrt{2} m_\lambda}{F} \int \prod_i^3 d^4 p_i \times \tilde{\lambda}(p_1) \sigma^{\mu\nu} \tilde{\chi}(p_2) \tilde{A}_\mu(p_3) (2\pi)^4 \delta^4\left(\sum_i^3 p_i\right) \times \left[1 - \frac{1}{6M^2} (p_1^2 + p_1 \cdot p_2 + p_2^2) \right] (p_1 + p_2)_\nu, \quad (18)$$

where m_λ is the one-loop gaugino mass [4]

$$m_\lambda = \frac{2e^2 N_q F_S}{16\pi^2 \langle S \rangle}. \quad (19)$$

Converting this to the coordinate space we obtain the Goldberger–Treiman vertex for $A_\mu\lambda\chi$ and its higher dimensional operator correction:

$$\mathcal{A} = \int d^4 x \mathcal{L}_{\chi\lambda A_\mu}(x), \quad (20)$$

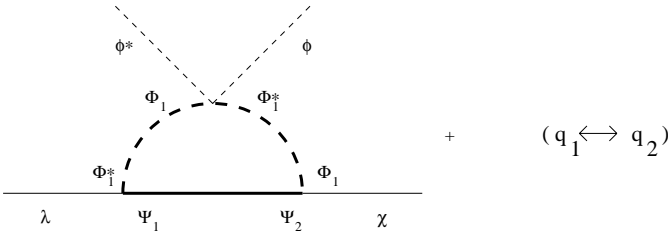


Fig. 3. Diagrams that give rise to the $\chi\lambda\phi^*\phi$ Goldberger–Treiman vertex. The thick solid and dashed lines denote fermionic and bosonic mediators, respectively. q_1, q_2 denote mediators

with

$$\mathcal{L}_{\chi\lambda A_\mu} = \frac{im_\lambda}{\sqrt{2}F} \quad (21)$$

$$\times \left[\chi\sigma^{\mu\nu} \left(1 + \frac{1}{6M^2} (\overleftarrow{\partial}^2 + \overleftarrow{\partial} \cdot \overrightarrow{\partial} + \overrightarrow{\partial}^2) \right) \lambda \right] F_{\mu\nu}.$$

Note that no on-shell condition for the goldstino was used in deriving (21).

The other dim-5 vertex of $\phi^*\phi\lambda\chi$ in the Goldberger–Treiman interaction arises from the two diagrams in Fig. 3. A straightforward calculation of the diagrams gives

$$i \frac{2\sqrt{2}he^3 N_q M F_S}{F} \int \prod_i^3 d^4 p_i \quad (22)$$

$$\times \tilde{\lambda}(p_1) \tilde{\chi}(p_2) \widetilde{\phi^* \phi}(p_3) (2\pi)^4 \delta^4 \left(\sum_i^3 p_i \right) I(p_1, p_2, M),$$

where

$$I(p_1, p_2, M) = \frac{i}{16\pi^2} \int \prod_i^3 dx_i \quad (23)$$

$$\times \delta \left(\sum_1^3 x_i - 1 \right) \left[\frac{1}{M^2 - t^2 + x_1 p_1^2 + x_2 p_2^2} \right].$$

Expanding in $1/M^2$ the denominator of the integrand in I and integrating over x_i we obtain the Goldberger–Treiman vertex and its correction:

$$\mathcal{A}_{\phi^* \phi \lambda \chi} = \int d^4 x \mathcal{L}_{\phi^* \phi \lambda \chi}(x), \quad (24)$$

with

$$\mathcal{L}_{\phi^* \phi \lambda \chi} = -\frac{em_\lambda}{\sqrt{2}F}$$

$$\times \left[\chi \left(1 + \frac{1}{6M^2} (\overleftarrow{\partial}^2 + \overleftarrow{\partial} \cdot \overrightarrow{\partial} + \overrightarrow{\partial}^2) \right) \lambda \right] \phi^* \phi. \quad (25)$$

Note that the higher dimensional operator corrections in (21) and (25) are independent of the number of the mediators N_q .

For non-abelian gauge theory, the corrections to the Goldberger–Treiman vertices can be found in essentially

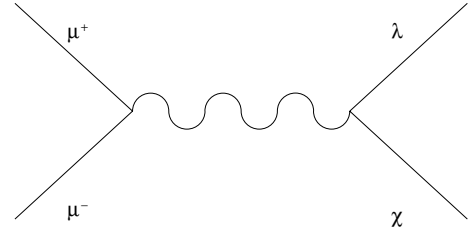


Fig. 4. Diagram for $\mu^- \mu^+ \rightarrow \chi\lambda$

the same way as in the abelian case. For SUSY QCD, for example, the Goldberger–Treiman vertices including the higher dimensional operator corrections for the dim-5 operators are given by

$$\mathcal{L}_{\text{GT}}^{\text{QCD}} = \frac{m_\phi^2 - m_\psi^2}{F} \chi \psi_i \phi_i^*$$

$$+ \frac{im_\lambda}{\sqrt{2}F} \left[\chi\sigma^{\mu\nu} \left(1 + \frac{1}{6M^2} (\overleftarrow{\partial}^2 + \overleftarrow{\partial} \cdot \overrightarrow{\partial} + \overrightarrow{\partial}^2) \right) \lambda^a \right] F_{\mu\nu}^a$$

$$- \frac{gm_\lambda}{\sqrt{2}F} \left[\chi \left(1 + \frac{1}{6M^2} (\overleftarrow{\partial}^2 + \overleftarrow{\partial} \cdot \overrightarrow{\partial} + \overrightarrow{\partial}^2) \right) \lambda^a \right] \phi_i^* T_{ij}^a \phi_j$$

$$+\text{h.c.}, \quad (26)$$

where g is the gauge coupling and T_{ij}^a are the gauge group generators.

These higher dimensional operator corrections can affect the goldstino production rate at high energy scattering near the threshold of the mediator particles. Consider, for example, the $\mu^- \mu^+ \rightarrow \chi\lambda$ process studied in [7–9]. The dominant amplitude for the process comes from the diagram in Fig. 4. The cross section to $O(s/M^2)$ from this diagram using the interaction (21) is

$$\sigma = \sigma_0 \left(1 + \frac{s}{6M^2} \right), \quad (27)$$

where

$$\sigma_0 = \frac{e^2 m_\lambda^2}{24\pi F^2}, \quad (28)$$

and s is the c.m. energy squared. This shows that, at $s^{1/2} = M$, for example, the goldstino production rate is increased by about 17% compared to that obtained without the higher dimensional operator correction. Of course, it would be very challenging to observe the direct production of goldstinos in GMSB models since in these models $F^{1/2}$ is generally too large for accelerator access. However, in models in which the SUSY breaking scale is accessible to accelerators, the higher dimensional operator corrections to the Goldberger–Treiman vertices could be used in probing the underlying SUSY breaking mechanism.

The corrections studied here can also have an important consequence in gravitino cosmology in GMSB models. The fact that the goldstino–matter interaction arises from loop diagrams indicates that goldstinos decouple from the MSSM fields at energies above the mediator mass. This becomes clear from (10), (15) and (23) which show that at high energies the goldstino–matter couplings decrease in proportion to M^2/E^2 , where E denotes the energy scale of

the process in consideration, compared to the Goldberger–Treiman vertices. This also implies that in the early universe light gravitinos decouple linearly in M^2/T^2 from the MSSM fields at a temperature higher than the mediator mass. It is therefore clear that the Goldberger–Treiman vertices are valid only below the mediator scale, and cannot be used at energies higher than the mediator mass. However, this fact has not been taken into account in the existing bound on the reheating temperature obtained from the gravitino overproduction [10]. When the decoupling is taken into account, one can expect that gravitino production at temperatures above the mediator scale is mostly due to the mediators whereas the MSSM fields contribution to the gravitino production is highly suppressed. This issue is currently under investigation [11].

Acknowledgements. The author is grateful to K. Choi and H.B. Kim for useful conversation, and owes special thanks to Guo-Hong Wu for useful discussions at the initial stage of the work. This work was supported in part by the Korean Science and Engineering Foundation.

References

1. P. Fayet, Phys. Lett. B **70**, 461 (1977); Phys. Lett. B **86**, 272 (1979)
2. R. Casalbuoni, S. De Curtis, D. Dominici, F. Feruglio, R. Gatto, Phys. Lett. B **215**, 313 (1988)
3. T.E. Clark, S.T. Love, Phys. Rev. D **54**, 5723 (1996)
4. M. Dine, A.E. Nelson, Phys. Rev. D **48**, 1277 (1993); M. Dine, A.E. Nelson, Y. Shirman, *ibid.* **51**, 1362 (1995); M. Dine, A.E. Nelson, Y. Nir, Y. Shirman, *ibid.* **53**, 2658 (1996)
5. T. Lee, G.-H. Wu, Phys. Lett. B **447**, 83–88 (1999)
6. J. Wess, J. Bagger, Supersymmetry and supergravity, 2nd ed. (Princeton University Press, Princeton, NJ 1992)
7. P. Fayet, Phys. Lett. B **175**, 471 (1986)
8. D. Dicus, S. Nandi, J. Woodside, Phys. Lett. B **258**, 231 (1991)
9. J.L. Lopez, D.V. Nanopoulos, A. Zichichi, Phys. Rev. Lett. **77**, 5168 (1996); Phys. Rev. D **55**, 5813 (1997)
10. T. Moroi, H. Murayama, M. Yamaguchi, Phys. Lett. B **303**, 289 (1993); Andre de Gouvea, T. Moroi, H. Murayama, Phys. Rev. D **56**, 1281 (1997)
11. K. Choi, H. B. Kim, T. Lee, in preparation